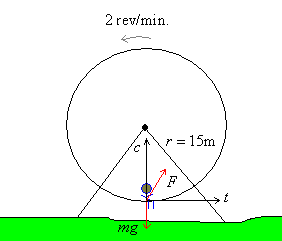
**Example: Ferris Wheel**

Consider a person (m = 70kg) rotating on a Ferris wheel, at a rate of 2 rev. per minute, and with a diameter of 30m. What is the force exerted on the person when they’re at the bottom of the Ferris wheel?



We label the Ferris wheel, the person, the centripetal/tangential axes, and the forces acting on him. The force, F, exerted by the seat will be going straight up we’ll find. But supposing we didn’t know that, we’ll show it going at an angle. So now, to find the force, F, we write out N2L. We’ll need the velocity of the person going in the circle, and so we’ll note that he makes one complete revolution (d = 2πr) every Δt = 30s, and so his velocity is:



So then,

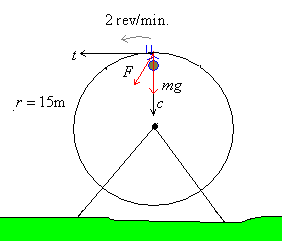


We used the fact that at = 0 b/c the speed is not changing with time. Solving for Fc now, we have,



**Example: Ferris Wheel**

Consider a person (m = 70kg) rotating on a Ferris wheel, at a rate of 2 rev. per minute, and with a diameter of 30m. What is the force exerted on the person when they’re at the top of the Ferris wheel?



We label the Ferris wheel, the person, the centripetal/tangential axes, and the forces acting on him. The force, F, exerted by the seat we’ll show going at an angle again. So now, to find the force, F, we write out N2L. So then,



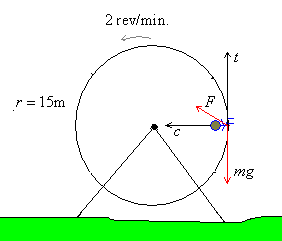
We used the fact that at = 0 b/c the speed is not changing with time. Solving for Fc now, we have,



So this indicates that our force goes in the negative **c** direction (this is upwards) and has a magnitude of 640N.

**Example: Ferris Wheel**

Consider a person (m = 70kg) rotating on a Ferris wheel, at a rate of 2 rev. per minute, and with a diameter of 30m. What is the force exerted on the person when they’re at the side of the Ferris wheel?



We label the Ferris wheel, the person, the centripetal/tangential axes, and the forces acting on him. The force, F, exerted by the seat we’ll show going at an angle again. So now, to find the force, F, we write out N2L. So then,



We used the fact that at = 0 b/c the speed is not changing with time. Now we see that the force has a centripetal and tangential component. Solving for these we get,



So our force is:



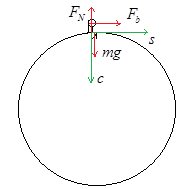
and it has a magnitude of:



and makes an angle of:



2a. You are riding in a Ferris wheel again. The wheel has a radius of 50m. You have a mass of 65kg. And it is rotating counter-clockwise at a rate of 0.15 rad/s. Suppose at a particular instant you are at the top of the wheel. We want to determine the force the bottom (FN) and back (Fb) of the seat exerts on you. Draw the wheel and you and the forces acting on you (there are 3 as in class). Then overlay this diagram with the s and c axes.



2b. Calculate the two forces F­N, and Fb using N2L in the s and c directions.

We have:

We do not know vs yet. So let’s get that. From the kinematics formulas we have vs = rω = (50)(0.15) = 7.5 m/s. Filling this into our formula for FN we get:



3. You are riding in a Ferris wheel again. The wheel has a radius of 50m. You have a mass of 65kg. Consider your at the same position as before, at the top of the Ferris wheel. And suppose you aren’t wearing a seat belt on this Ferris wheel (bad idea). A maniacal Ferris wheel operator red lines the Ferris wheel motor, causing the Ferris wheel to rotate faster and faster. Eventually it will reach a rotational velocity great enough to cause you to fly off the seat (if you aren’t holding on to it). What is this rotational velocity ω? (Hint: what will FN be when you are about to come off the seat?)

Repeating our analysis, we have:

This time FN = 0 so,



This translates to the following rotation velocity:



**Question 7**. You (m = 72kg) are riding a Ferris wheel which rotates at a constant rate of 2 rev/min. The radius of the Ferris wheel is R = 32m. What force does your seat exert on you at the top of the ride?

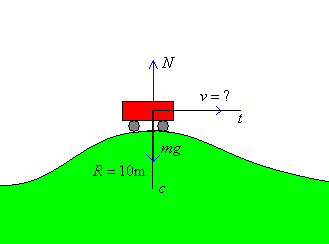


**Question 4**. You (m = 80kg) are riding a Ferris wheel which rotates at a constant rate of 1.4 rev/min. The radius of the Ferris wheel is R = 27m. What force does your seat exert on you at the top of the ride?



**Example: How fast can you go over a hill with radius of curvature R = 10m without flying off?**

Consider the diagram below, with forces labeled, and axes drawn.



So write out N2L using this fact, and solve for v.

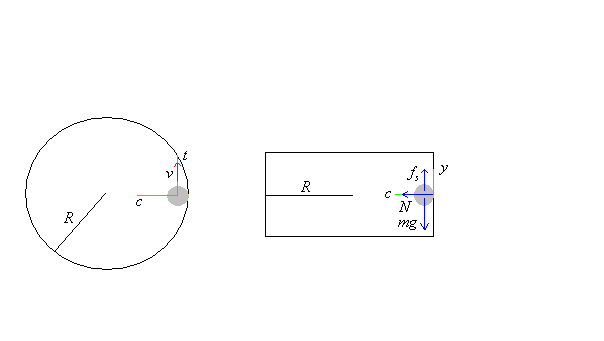


As you may know by experience, the faster you go over such a hill, the more your car ‘lifts’ off of the ground. This means that the force N is reduced. At the speed v, at which your car loses contact with the ground, N will be 0. Therefore we have,



**Example: Cyclotron**.

Assume the coefficient of static friction between you and the wall of a ‘gravitron’ is 0.8, and that the gravitron has a radius of 5m. How fast must it revolve in order for you to be pinned to the wall? Give the rate of revolution in rev/s.



I’ve drawn the situation in two ways. The first is top down with the gravitron rotating CCW, and the second is a side view with the right hand side of the gravitron rotating into the page. In this case, we have no tangential forces – only forces in the c and y directions. But now, let’s add the forces,



In this line we note that we’re looking for the minimum rate of rotation necessary, this will be achieved when fs is at its maximum → fs = μsN. Also, we observe that the person is not accelerating in the y direction – they should be completely stationary in that direction.



Equating these two we have,



Now we need the rate of rotation. Since a revolution is made after the person travels a distance of C = 2πR, the rate of revolution, or frequency, is:



Therefore a revolution must be made every T = 1/f = 4s.

**Example**

Suppose you connect a mass mass (m = 5kg) to a spring (equilibrium length ℓ =1m, k = 500N/m), and swing it around around in a circle on a table at a speed of one revolution every 1.2 seconds. What is the stretch of the spring? What is the speed of the mass?

Let x be the stretch of the spring. Then, adding up the forces in the centripetal direction we have,



Additionally, the velocity is given by:



So filling this into the top equation we have,

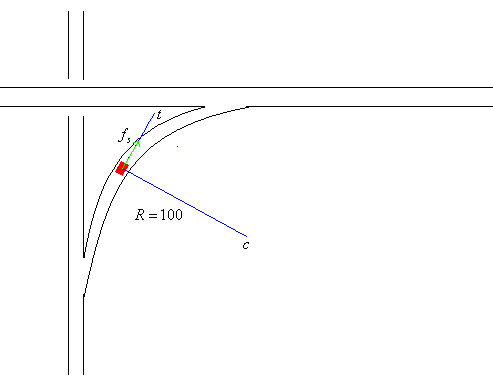


So the spring will stretch 38cm.

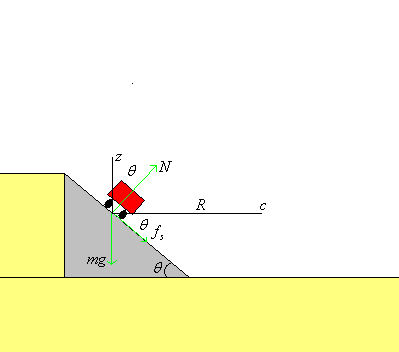
**Example: Banking angle θ, needed to keep car from slipping when on on-ramp.**

Suppose you’re a civil engineer designing an on-ramp onto an interstate. As you may note, on-ramps are banked to facilitate accelerating onto the highway, and they are also, necessarily, curved. You want to design an on-ramp with the minimum necessary banking angle to allow a car to accelerate from 40mph (18m/s) to 70mph (31m/s). Suppose that the coefficient of static friction between rubber tires and the road is μs = 0.95. And also suppose that the on-ramp must have a radius of curvature of 100m.

Alright, the situation is illustrated from two different perspectives. The first is the top-down perspective. The radius of curvature of the on-ramp is denoted R. Note that a static friction force fs acts on the car pushing it forward (recall that the static friction force pushes the car forward because the tires on the car rotate such that the bottom surface of the tire pushes back against the road, which causes friction from the road to push forward against the tire, and hence the car, accelerating it forward).



The side view is this. the t axis is going into the screen, and the car is pictured going into the screen as well. The road is banked at an angle θ. As usual N and mg act on the car, and friction force fs acts on the car pushing it downward, because the tendency will be for the car to slide up the road. θ is also the angle between fs and c, as well as N and z – as you may verify.



Now having labeled all forces acting on the car, we simply write out N2L in the z, c directions.



Now since we’re interested in the minimum necessary angle, we can assume fs = μsN – its maximum value. Also, since we need to the road to accommodate speeds up to 31m/s, we will let v = vmax = 31m/s. Additionally, observe that az = 0 because we do not want the car to be going up or down. So these considerations simplify everything to:



We can solve this system of equations…Solve for Nx first and plug into the second equation:



And now solving for Nx,



thus we have,

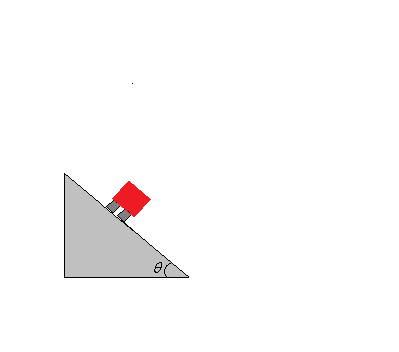


one can divide these two into each other to get θ,



so we don’t have to tilt it very much as we see – less than a degree. If the radius of curvature of the on-ramp were shorter, then we’d have to tilt the road more.

1. A car turns onto an on-ramp, angled at θ with respect to the ground, and which has a radius of 175m. If the car’s speed is 27m/s, what minimum banking angle is required in order to keep the car from sliding? Suppose friction is negligible. Note the car is traveling *into* the page from this perspective.



Applying N2L in the upward direction we have:



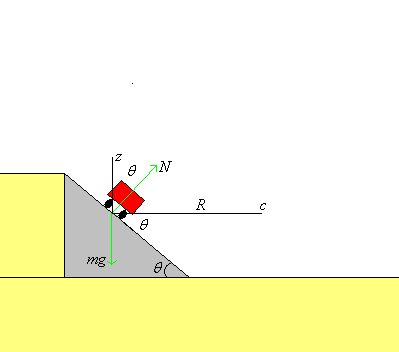
And in the centripetal direction,



Plugging the former into the latter we have:



3. A car turns onto an on-ramp, angled at θ with respect to the ground, and which has a radius of 150m. If the car’s speed is 30m/s, what minimum banking angle is required in order to keep the car from sliding. Suppose friction is negligible.



The forces are shown. Applying N2L in the centripetal and vertical directions…

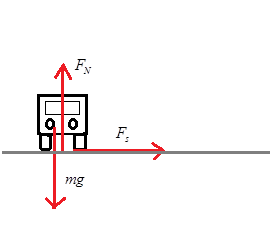


we find that,



**10**. A car rounds a turn (radius of curvature of turn is r = 75m). If the coefficient of static friction between the tires and the road is μs = 1.2, how quickly can the car round the turn without slipping?

A diagram of the forces is given below. The perspective is that the car is rounding the curve, coming out of the page towards you. So the center of the circle it is going around is to the right of the car. The static friction force points to the right because as the car rounds the turn, it would tend to slip to the left, and so friction will oppose this tendency by pushing to the right.



N2L reads.



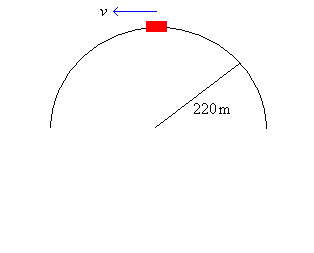
Now we’re looking for the maximum speed with which the car can round the turn. This will happen when the static friction force is at its maximum value Fs = μsFN = μsmg. So let’s plug that into the top equation:



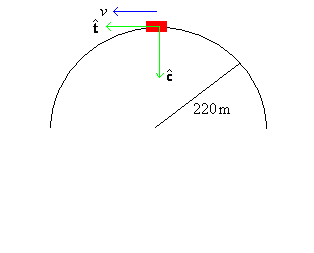
**Problem 4.26**

A car at the Indianapolis 500 accelerates uniformly from the pit area, going from rest to 320 kph in a semi-circular arc with a radius of 220m. Determine the tangential and radial acceleration of the car when it is halfway through the turn, assuming constant tangential acceleration. If the curve were flat, what would the coefficient of static friction have to be between the tires and the road to provide this acceleration with no slipping or skidding?

The situation is diagrammed below.



To faciliate our calculation of the acceleration we’ll overlay our centripetal and tangential axes.



Now



where we convert kph to m/s (there are 1000m in a km, and 3600s in an hour). Now the tangential acceleration is defined to be:



We’re not given Δt. But observe that at just the acceleration of the speed of the car around the track. We can use another equation for it. Imagine the track is straightened out into a line. The car has an initial speed of 0, and a final speed of 350kph = 350 (1000/3600) m/s = 97.2 m/s. And it travels through a distance of d = 2πr/4 = 2π(220)/4 = 346m (a quarter of a circumference). So we can get at via:



So then the magnitude of the total acceleration is:



The friction force must provide the force for this acceleration, and so according to N2L we must have,



Since we’re looking for the minimum coefficient of static friction that can accomplish this, we will allow Fs to be at its maximum value,



3. Suppose you’re driving a car along a track (radius r = 60m). Starting from rest you accelerate around the track with an acceleration at = 2 m/s2. If the coefficient of static friction between the tires and the track is μs = 1.3, at what time will the car start to slip? Hint: it will start to slip when the static friction force is no longer able to support the car’s total acceleration.

Friction will act as the centripetal force pushing the car towards the center of the circle, keeping it from slipping. So we’ll have,



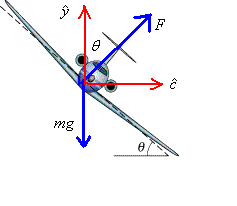
and in the tangential direction we have:



As the car goes faster, the friction force will increase to keep it on the track. When the friction force is at its maximum value, Fs = μsFN, the car will begin to slip. This velocity will be at,



2. An airplane is flying in a horizontal circle at a speed of 400 km/h (see the figure). If its wings are tilted at angle *θ* = 30° to the horizontal, what is the radius of the circle in which the plane is flying? Assume that the required force is provided entirely by an “aerodynamic lift” that is perpendicular to the wing



N2L in centripetal direction is:



In y-direction:



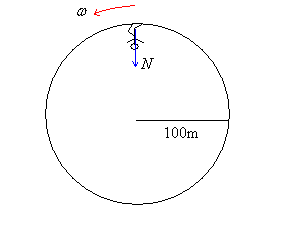
Dividing two equations we get:



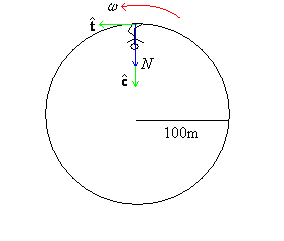
**Problem 4.13**

At what minimum speed must a roller coaster be traveling when upside down at the top of a circle so that passengers will not fall out. Assume a radius of curvature of 7.4m.

Our situation is diagrammed below, with the normal force acting on you, and the unknown rate of rotation, ω labelled.



There are no other forces acting on you because there is no gravity since we’re out in space. If the cylinder isn’t rotating, then N will be 0 because you’ll just fall off of the floor. But, like with the ‘gravitron’ at a fair, if we start the cylinder spinning, then you’ll pick up a centripetal acceleration, and the wall normal force will exert a centripetal force on you, i.e., N won’t be 0, and you won’t fall off. In order for us to feel a simulated gravitational force, we would need N to be equal to your weight on Earth, W = mg. So overlaying our centripetal axes, and applying N2L,





So the tangential speed necessary is:



This means that you’d be going 9.9m/s if you stand on the floor. Now we need to determine how many revolutions per minute are needed. Let’s determine the time for one revolution. This would be:

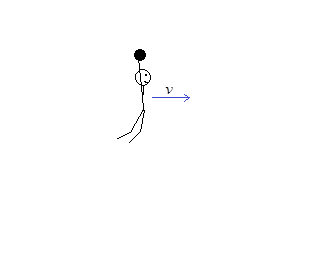


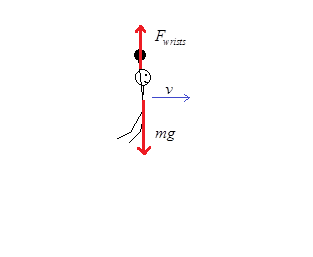
And so the number of revolutions per second would be,



**Problem 2**

A gymnast on the high bars rotates around the bar with a speed of v = 5m/s. If her mass is m = 55kg, what tension must be in her wrists for her to hold onto the bar? Take the distance between the bar and her center of mass to be r = 0.75m. Don’t forget to include the force of gravity. And note this is just a circular motion problem (no torque, moment of inertia, etc., stuff needed).

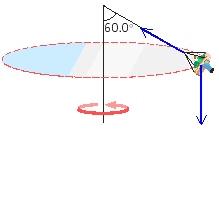




The gymnast experiences too forces, drawn above. According to N2L we have:



1. A "swing" ride at a carnival consists of chairs that are swung in a circle by 25 m cables attached to a vertical rotating pole, as the drawing shows. Suppose the total mass of a chair and its occupant is 150 kg. Find the speed of the chair.



Applying N2L in the y-direction we have:



And in the centripetal direction,



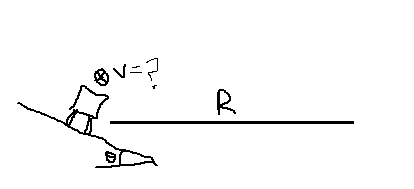
or suppose you know angle and angular velocity. Can determine speed?



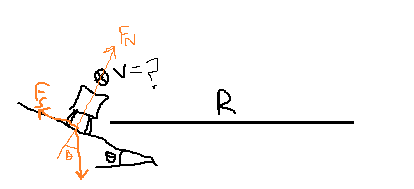
And in the centripetal direction,



**Question 5.** Suppose that a car is driving in a circular path (R = 120m) along an embankment. Suppose the embankment is angles at θ = 10°. If the coefficient of static friction between the tires and road is μs = 0.1, what is the *minimum* speed the car may have w/o sliding *down* the embankment? Hint: consider which way the friction force is pointing in this case, if you tend to slide *down*.



Drawing forces we have



Adding forces in the y direction we have:



In centripetal direction, and accounting for fact that Fs is at its max value as we’re ‘just about to slip’, we have:

